

Hierarchy sensitivity of $\text{NO}\nu\text{A}$ in light of T2K ν_e appearance data

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Abstract

The ν_e appearance data of T2K experiment has given a glimpse of the allowed parameters in the hierarchy- δ_{CP} parameter space. In this paper, we explore how this data affects our expectations regarding the hierarchy sensitivity of the $\text{NO}\nu\text{A}$ experiment. For the favourable combinations of hierarchy and δ_{CP} , the hierarchy sensitivity of $\text{NO}\nu\text{A}$ is unaffected by the addition of T2K data. For the unfavourable combinations, $\text{NO}\nu\text{A}$ data gives degenerate solutions. Among these degenerate solutions, T2K data prefers IH and δ_{CP} in the lower half plane over NH and δ_{CP} in the upper half plane. Hence, addition of the T2K data to $\text{NO}\nu\text{A}$ creates a bias towards IH and δ_{CP} in the lower half plane irrespective of what the true combination is.

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I. INTRODUCTION

Discovery of neutrino oscillations has led to an explosion of interest in understanding the fundamental properties of neutrinos. With the data from the solar and atmospheric neutrino experiments, we have a picture of three neutrino flavours, ν_e , ν_μ and ν_τ , mixing with one another to form three light neutrino mass eigenstates ν_1 , ν_2 and ν_3 . Measurement of the survival probability of electron neutrinos in the solar neutrino experiments [1, 2] and that of electron anti-neutrinos in KamLAND [3, 4] led to a precise determination of $\Delta_{21} = m_2^2 - m_1^2$ and θ_{12} . Measurement of the muon neutrino survival probability by the MINOS [5] and T2K [6] experiments led to the precise determination of $\sin^2 2\theta_{23}$ and $|\Delta m_{\mu\mu}^2|$. The data indicates that the two mixing angles θ_{12} and θ_{23} are quite large (in fact, θ_{23} is close to maximal) [7] and $\Delta_{21} \ll |\Delta m_{\mu\mu}^2|$. The values of $\Delta_{31} = m_3^2 - m_1^2$ and $\Delta_{32} = m_3^2 - m_2^2$ can be obtained from the relation [8]

$$\Delta m_{\mu\mu}^2 = \sin^2 \theta_{12} \Delta_{31} + \cos^2 \theta_{12} \Delta_{32} + \cos \delta_{\text{CP}} \sin 2\theta_{12} \sin \theta_{13} \tan \theta_{12} \Delta_{21}. \quad (1)$$

At present only the magnitude of $\Delta m_{\mu\mu}^2$ is known but not its sign. Since $\Delta_{21} \ll |\Delta m_{\mu\mu}^2|$, the signs of Δ_{31} and Δ_{32} are the same as that of $\Delta m_{\mu\mu}^2$. If Δ_{31} is positive, a likely neutrino mass pattern is $m_3 \gg m_2 > m_1$, which is called normal hierarchy (NH). If Δ_{31} is negative, the neutrino mass pattern is likely to be $m_2 > m_1 \gg m_3$, which is called inverted hierarchy (IH). It is of course possible to have Δ_{31} positive or negative when all the three neutrino masses are quasi-degenerate. In such a situation also, positive Δ_{31} is called NH and negative Δ_{31} is called IH.

In the past few years, reactor neutrino experiments DoubleCHOOZ, Daya Bay and RENO [9–11], with baselines ~ 1 km, have measured θ_{13} to be non-zero. The moderately large value of θ_{13} has given hope that the outstanding questions related to neutrino oscillations can soon be answered. These questions are

- What is correct neutrino mass hierarchy, NH or IH?
- What is the true octant of θ_{23} ? Is $\theta_{23} < \pi/4$ or $> \pi/4$?
- Is there CP violation in the neutrino sector? If yes, what is the value of the CP violating phase δ_{CP} ?

All these questions can be answered by the measurement of the oscillation probabilities $P(\nu_\mu \rightarrow \nu_e)$ and $P(\bar{\nu}_\mu \rightarrow \bar{\nu}_e)$ at the long baseline neutrino experiments T2K and NO ν A. T2K experiment has already taken significant amount of data and NO ν A experiment has begun its run. In this paper, we address the question: How does the data of T2K modify our expectations regarding the mass hierarchy determination capability of NO ν A?

II. HIERARCHY- δ_{CP} DEGENERACY

The oscillation probabilities $P(\nu_\mu \rightarrow \nu_e)$ and $P(\bar{\nu}_\mu \rightarrow \bar{\nu}_e)$ can be calculated in terms of the three mixing angles θ_{12} , θ_{13} and θ_{23} , the mass-squared differences Δ_{21} and Δ_{31} and the CP violating phase δ_{CP} . In long baseline experiments, however, the neutrinos travel long distances through earth matter and undergo coherent forward scattering. The effect of this scattering is taken into account through the Wolfenstein matter term [12]

$$A \text{ (in eV}^2\text{)} = 0.76 \times 10^{-4} \rho \text{ (in gm/cc)} E \text{ (in GeV)}, \quad (2)$$

where E is the energy of the neutrino and ρ is density of the matter. The interference between A and Δ_{31} leads to the modification of neutrino oscillation probability due to matter effects. The expression for $P(\nu_\mu \rightarrow \nu_e)$ is given by [13, 14]

$$\begin{aligned} P(\nu_\mu \rightarrow \nu_e) = & \sin^2 2\theta_{13} \sin^2 \theta_{23} \frac{\sin^2 \hat{\Delta}(1 - \hat{A})}{(1 - \hat{A})^2} \\ & + \alpha \cos \theta_{13} \sin 2\theta_{12} \sin 2\theta_{13} \sin 2\theta_{23} \cos(\hat{\Delta} + \delta_{\text{CP}}) \frac{\sin \hat{\Delta} \hat{A} \sin \hat{\Delta}(1 - \hat{A})}{\hat{A} (1 - \hat{A})} \\ & + \alpha^2 \sin^2 2\theta_{12} \cos^2 \theta_{13} \cos^2 \theta_{23} \frac{\sin^2 \hat{\Delta} \hat{A}}{\hat{A}^2}. \end{aligned} \quad (3)$$

where $\hat{\Delta} = \Delta_{31}L/4E$, $\hat{A} = A/\Delta_{31}$ and $\alpha = \Delta_{21}/\Delta_{31}$. For anti-neutrinos, $P(\bar{\nu}_\mu \rightarrow \bar{\nu}_e)$ is given by a similar expression with $\delta_{\text{CP}} \rightarrow -\delta_{\text{CP}}$ and $A \rightarrow -A$.

$P(\nu_\mu \rightarrow \nu_e)$ is sensitive to the neutrino mass hierarchy because both $\hat{\Delta}$ and \hat{A} change sign under a change of sign of Δ_{31} . The term $\sin[(1 - \hat{A})\hat{\Delta}]/(1 - \hat{A})$ undergoes a change under the sign change of \hat{A} . This change may or may not be measurable because value of δ_{CP} is completely unknown at the moment. For certain choices of hierarchy and values of δ_{CP} , the change in the first term of eq. (3) arising due to changing the hierarchy can be compensated by a change in the second term caused by choosing a wrong value of δ_{CP} . It

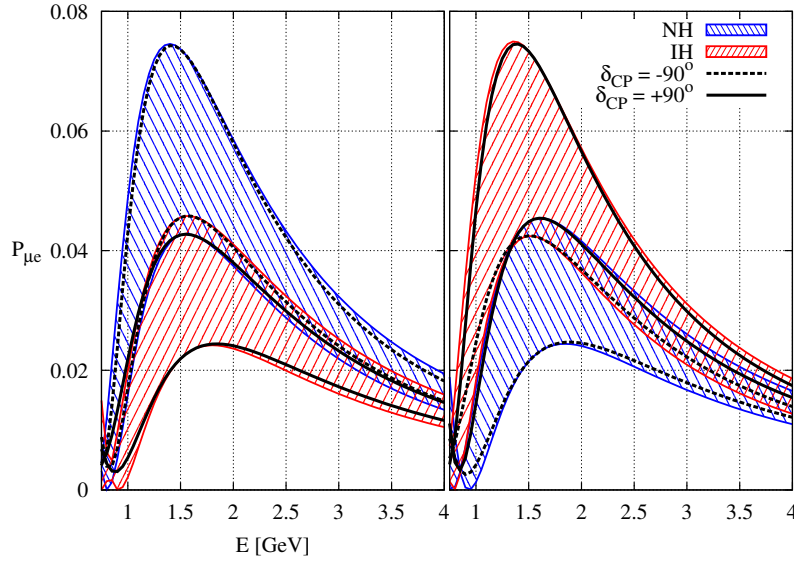


FIG. 1: $P(\nu_\mu \rightarrow \nu_e)$ (left panel) and $P(\bar{\nu}_\mu \rightarrow \bar{\nu}_e)$ (right panel) vs. energy for NO ν A. Variation of δ_{CP} leads to the blue (red) bands for NH (IH). The plots are drawn for maximal θ_{23} and other neutrino parameters given in the text.

was shown that NO ν A experiment [15] can determine the neutrino mass hierarchy, that is measure the change induced by the matter term, for the following two favourable cases:

- hierarchy is NH and δ_{CP} is in the lower half plane ($-180^\circ \leq \delta_{\text{CP}} \leq 0$) and
- hierarchy is IH and δ_{CP} is in the upper half plane ($0 \leq \delta_{\text{CP}} \leq 180^\circ$).

If nature had chosen either of these favourable cases, NO ν A can determine both the hierarchy and the half plane of δ_{CP} . For the two unfavourable cases,

- hierarchy is NH and δ_{CP} is in the upper half plane ($0 \leq \delta_{\text{CP}} \leq 180^\circ$) and
- hierarchy is IH and δ_{CP} is in the lower half plane ($-180^\circ \leq \delta_{\text{CP}} \leq 0$),

an analysis of NO ν A data gives degenerate solutions. Hence NO ν A alone is unable to determine the hierarchy for all possible combinations of hierarchy and δ_{CP} [16, 17]. This is illustrated in the plots of $P(\nu_\mu \rightarrow \nu_e)$ and $P(\bar{\nu}_\mu \rightarrow \bar{\nu}_e)$ for NO ν A, shown in fig. 1. In this paper, we study how the presently collected neutrino data from the T2K [6, 18] modifies these expectations from NO ν A.

III. SIMULATION AND CALCULATION DETAILS

We have mentioned in the previous section that NO ν A can determine the hierarchy by itself, for favourable hierarchy- δ_{CP} combinations. It was suggested that a combination of data from NO ν A and T2K may be able to determine the hierarchy for unfavourable combinations also [16, 17]. Since T2K has already produced about one year of neutrino data, we now explore the hierarchy determination capability of NO ν A in light of this data.

A difficulty arises in combining the simulations of NO ν A with the data of T2K. The data of T2K contain random fluctuations but the simulations of NO ν A do not. For data without fluctuations, χ^2_{min} is zero whereas for data with fluctuations, χ^2_{min} is expected to be equal to the degrees of freedom. The question then arises: How to combine the simulations and data in such a way that we can generate practical definitions of χ^2_{min} and $\Delta\chi^2$ which can be used in analysis? The only practical approach is to simulate NO ν A data with fluctuations.

A. Simulation of NO ν A experiment

NO ν A [15] is a long baseline neutrino oscillation experiment capable of measuring the survival probability $P(\nu_\mu \rightarrow \nu_\mu)$ and the oscillation probability $P(\nu_\mu \rightarrow \nu_e)$. The NuMI beam at Fermilab, with the power of 700 kW which corresponds to 6×10^{20} protons on target (POT) per year, produces the neutrinos. The far detector consists of 14 kton of totally active scintillator material and is located 810 km away at a 0.8° off-axis location. Due to the off-axis location, the flux peaks sharply at 2 GeV, which is close to the energy of maximum oscillation of 1.4 GeV. It has started taking data in 2014 and is expected to run three years in neutrino mode and three years in anti-neutrino mode. In our simulations, we have taken the retuned signal acceptance and background rejection factors from [19, 20].

In doing the simulations, we have used the "true" values of the neutrino parameters to be their central values, namely $\sin^2 \theta_{12} = 0.3$, $\sin^2 2\theta_{13} = 0.084$, $\sin^2 \theta_{23} = 0.514$, $\Delta_{21} = 7.5 \times 10^{-5} \text{ eV}^2$ and $\Delta m_{\mu\mu}^2 = \pm 2.4 \times 10^{-3} \text{ eV}^2$ [6, 7]. The values for $\Delta_{31}(\text{NH})$ and $\Delta_{31}(\text{IH})$ were derived from Δm_{eff}^2 using the expression given in eq. (1). Simulations were done with NH as the true hierarchy as well as with IH. The following true values of δ_{CP} were chosen as inputs in the simulations: -135° , -90° , -45° , 0 , 45° , 90° , 135° and 180° . We used these true values as inputs in the software GLOBES [21, 22] to calculate the expected ν_e appearance

events in i th energy bin N_i^{exp} .

To take into account the possible fluctuations in the expected data, we took N_i^{exp} , and gave it as an input to the Poissonian random number generator code [23]. This code generated 100 Poissonian random numbers whose mean is N_i^{exp} . We repeated this procedure for all the energy bins. Thus, we generated 100 possible event numbers for each bin. We collected the first of the 100 numbers from i th energy bin and labelled it $N_i^{\text{data}\#1}$. By collecting the second of the 100 numbers from the i th energy bin we obtain $N_i^{\text{data}\#2}$ etc. Thus, we obtain 100 independent simulations of the ν_e appearance data which include the random Poissonian fluctuations expected in counting experiments.

The “theoretical” event rates, corresponding to this data, are calculated for various test values of the neutrino parameters. The test values for $\sin^2 2\theta_{13}$ ($\sigma(\sin^2 2\theta_{13}) = 5\%$) [7] and $\Delta m_{\mu\mu}^2$ ($\sigma(\Delta m_{\mu\mu}^2) = 3\%$) [24] are selected within the $\pm 2\sigma$ range of the central values. Since $\sin^2 \theta_{23}$ is not-so well constrained, its test values are picked within the $\pm 3\sigma$ range: [0.35, 0.65]. Test values of δ_{CP} spanned its total allowed range: $[-180^\circ, 180^\circ]$. With the selected test values as inputs to GLoBES, we calculated N_i^{test} for ν_e appearance as functions of the test values of neutrino parameters. As before, here i stands for the i th energy bin.

We compute the Poissonian χ^2 between $N_i^{\text{data}\#1}$ and N_i^{test} using the formula [25]

$$\chi^2(1) = \sum_i 2[(N_i^{\text{test}} - N_i^{\text{data}\#1}) + N_i^{\text{data}\#1} \times \ln(N_i^{\text{data}\#1}/N_i^{\text{test}})] + \sum_j [2 \times N_j^{\text{test}}] + \chi^2(\text{prior}) \quad (4)$$

where i stands for bins for which $N_i^{\text{data}\#1} \neq 0$ and j stands for bins for which $N_j^{\text{data}\#1} = 0$. $\chi^2(\text{prior})$ is the prior added due to the deviation of the test values of neutrino parameters from their best fit values. It is defined by

$$\begin{aligned} \chi^2(\text{prior}) = & ((\sin^2 2\theta_{13}(\text{test}) - 0.084)/(0.05 \times 0.084))^2 + \\ & ((\sin^2 2\theta_{23}(\text{test}) - 4 \times 0.514 \times 0.486)/(0.02 \times 4 \times 0.514 \times 0.486))^2 + \\ & (|\Delta m_{\mu\mu}^2(\text{test})| - 2.40 \times 10^{-3})/(0.03 \times 2.40 \times 10^{-3}) \end{aligned} \quad (5)$$

Since N_i^{test} is a function of the test values of the neutrino parameters, $\chi^2(1)$ is also a function of the same test values. We find the minimum value of $\chi^2(1)$ and subtract it from each of the values of $\chi^2(1)$ to obtain $\Delta\chi^2(1)$. It is zero for those test values of neutrino parameters for which $\chi^2(1)$ is minimum. Since $N_i^{\text{data}\#1}$ contains fluctuations, the test values of neutrino

parameters for which $\Delta\chi^2(1)$ vanishes are not the same as the input values used in the simulations.

Next we marginalize $\Delta\chi^2(1)$ over $\sin^2 2\theta_{13}$, $\sin^2 \theta_{23}$ and $|\Delta m_{\text{eff}}^2|$ but not over δ_{CP} and hierarchy and label the result $\Delta\chi_m^2(1)$. Thus $\Delta\chi_m^2(1)$ is a function of test δ_{CP} and test hierarchy. As mentioned above, $\Delta\chi_m^2(1)$ is zero for some value of test δ_{CP} and test hierarchy. We then compute $\Delta\chi_m^2(2)$ from $N_i^{\text{data}\#2}$ using the procedure described above. $\Delta\chi_m^2(2)$ also vanishes for some value of test δ_{CP} and test hierarchy but these values need not be the same ones for which $\Delta\chi_m^2(1)$ vanishes. Treating $N_i^{\text{data}\#p}$ ($1 \leq p \leq 100$) as the “data”, we compute 100 different sets of $\Delta\chi_m^2(p)$ as functions of test δ_{CP} and test hierarchy. Each of these sets contains a zero element at some test δ_{CP} and test hierarchy. However, for a given hierarchy and a given test value of δ_{CP} , a large number of $\Delta\chi_m^2(p)$ will be non-zero. We take the average of these 100 $\Delta\chi_m^2$ values to finally obtain $\overline{\Delta\chi_m^2}$ as a function of the test values of δ_{CP} and hierarchy. The quantity $\overline{\Delta\chi_m^2}$ is equivalent to the $\Delta\chi^2$ obtained in simulations where the “data” was simulated without fluctuations.

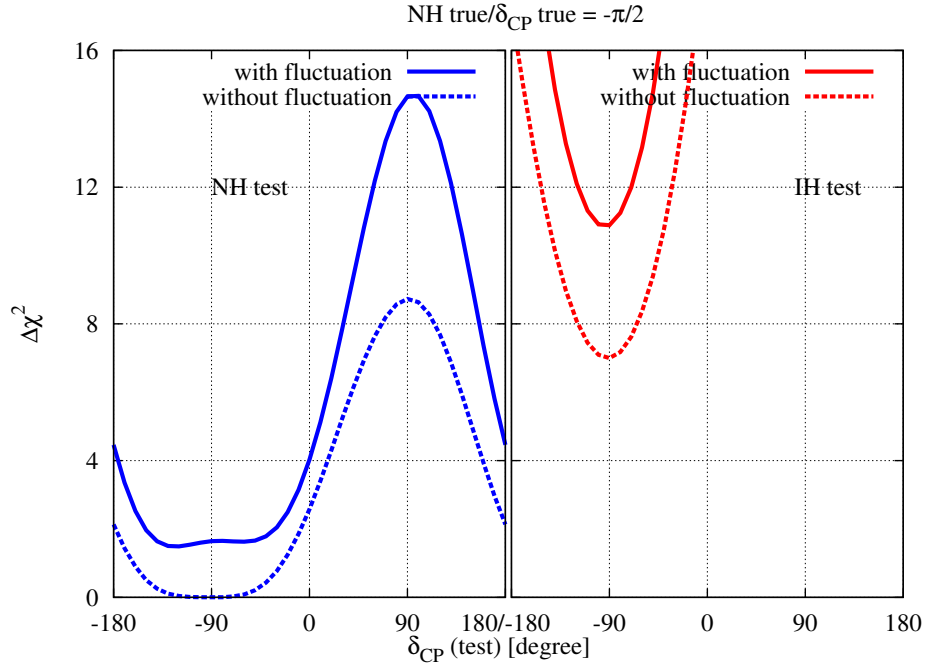


FIG. 2: Comparison between $\Delta\chi^2$ vs test values of δ_{CP} from NOνA simulation without fluctuations and with fluctuations. NH is the true hierarchy and true value of δ_{CP} is -90° .

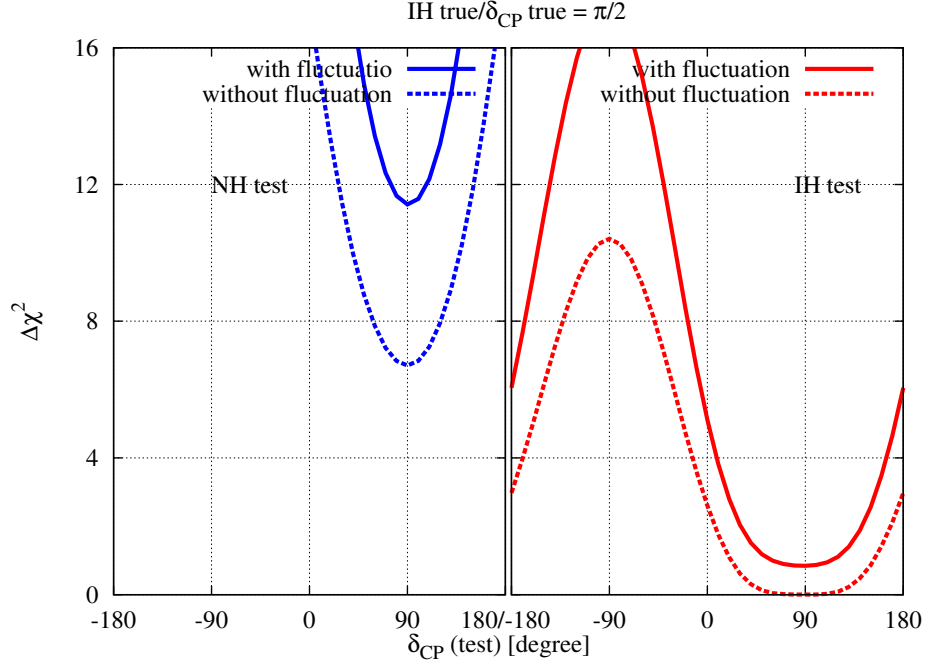


FIG. 3: Comparison between $\Delta\chi^2$ vs test values of δ_{CP} from NO ν A simulation without fluctuations and with fluctuations. IH is the true hierarchy and true value of δ_{CP} is 90° .

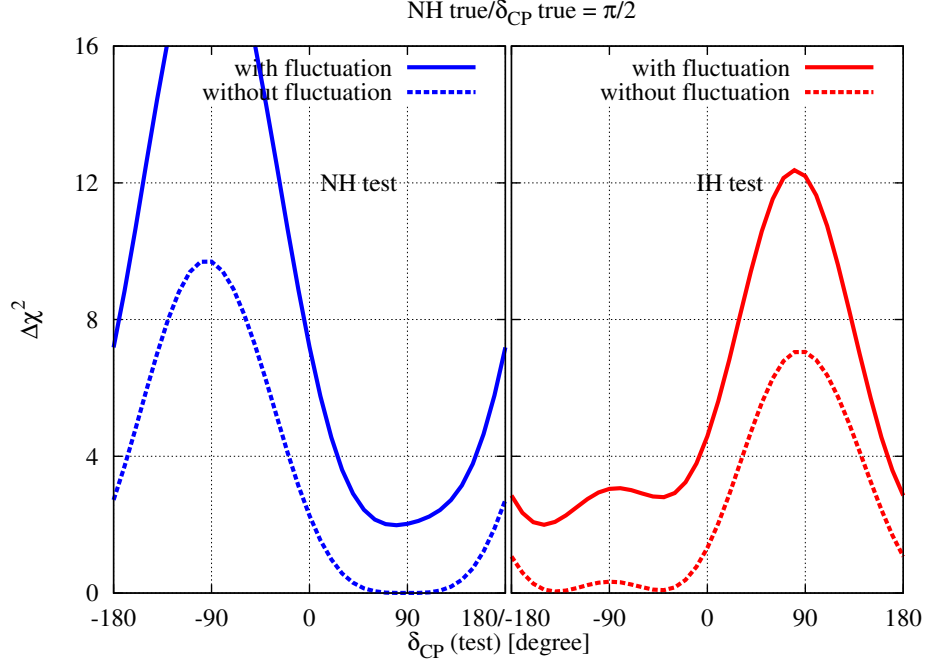


FIG. 4: Comparison between $\Delta\chi^2$ vs test values of δ_{CP} from NO ν A simulation without fluctuations and with fluctuations. NH is the true hierarchy and true value of δ_{CP} is 90° .

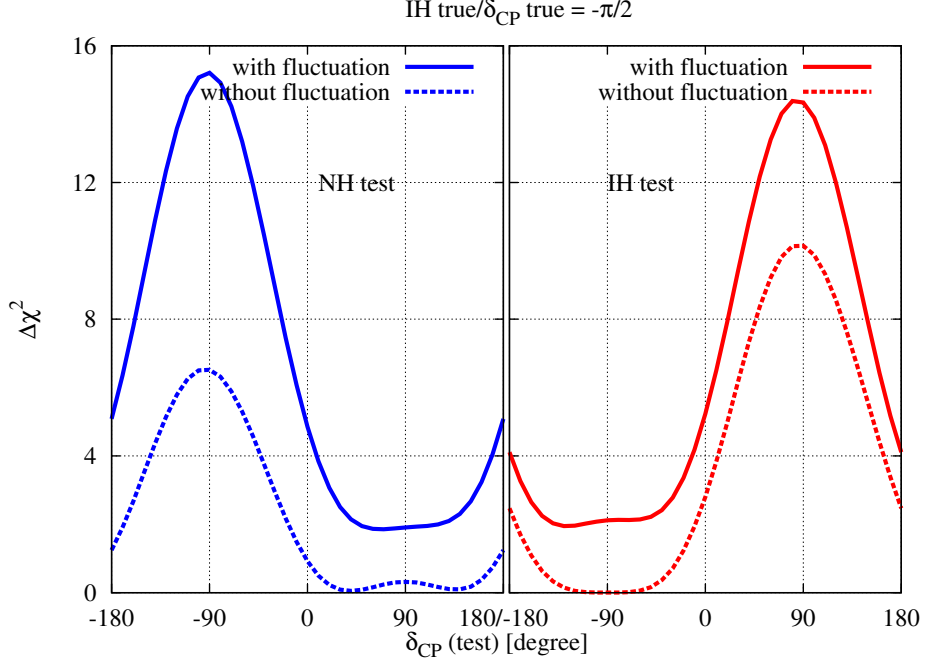


FIG. 5: Comparison between $\Delta\chi^2$ vs test values of δ_{CP} from NO ν A simulation without fluctuations and with fluctuations. IH is the true hierarchy and true value of δ_{CP} is -90° .

In figures 2 to 5, we have plotted the hierarchy-discriminating $\Delta\chi^2$ vs test δ_{CP} . All these figures contain two curves: One curve is obtained by our procedure of calculating $\overline{\Delta\chi_m^2}$ from NO ν A simulation with fluctuations and the other curve is obtained by doing simulations without fluctuations. The plots show hierarchy discrimination for the for two most favourable hierarchy - δ_{CP} combinations (NH and $\delta_{\text{CP}} = -90^\circ$ in fig. 2 & IH and $\delta_{\text{CP}} = 90^\circ$ in fig. 3) and two most unfavourable hierarchy - δ_{CP} combinations (NH and $\delta_{\text{CP}} = 90^\circ$ in fig. 4 & IH and $\delta_{\text{CP}} = -90^\circ$ in fig. 5). We see that for these four cases, $\overline{\Delta\chi_m^2}$ matches qualitatively with $\Delta\chi^2$. $\overline{\Delta\chi_m^2}$ never vanishes because of the averaged effect of the fluctuations but the physics remains same in simulations both with and without fluctuations. This verifies our earlier statement that $\overline{\Delta\chi_m^2}$ correctly represents the hierarchy sensitivity.

To check the stability of this averaging method, we have also done 1000 independent simulations of NO ν A. That is, we have generated 1000 random Poissonian event numbers for each bin, whose mean is equal to the event number of that bin. Then we followed the above procedure to calculate $\overline{\Delta\chi_m^2}$. In figure 6, we have compared the $\overline{\Delta\chi_m^2}$ from 100 independent simulations with that of 1000 independent simulations for IH and $\delta_{\text{CP}} = 90^\circ$. We see that the $\overline{\Delta\chi_m^2}$ s from both the simulations match quite closely. This holds true for other hierarchy- δ_{CP}

combinations as well. Thus the values of $\overline{\Delta\chi_m^2}$ s, derived by our simulation, are stable and we will use this method of 100 independent simulations to determine the hierarchy sensitivity of NO ν A after adding present T2K data.

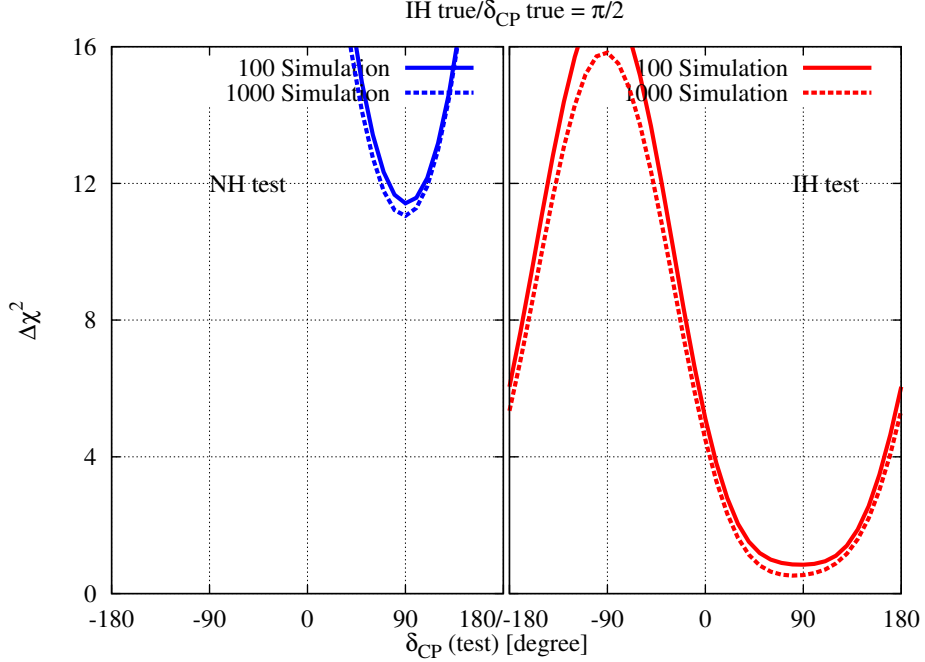


FIG. 6: Comparison between $\Delta\chi^2$ vs test values of δ_{CP} from NO ν A simulation with fluctuations for 100 independent simulations and 1000 independent simulations. IH is the true hierarchy and true value of δ_{CP} is 90° .

B. T2K Calculation

T2K is a long baseline neutrino oscillation experiment with the ν_μ beam from the J-PARC accelerator in Tokai to the Super-Kamiokande detector 295 km away. The accelerator is oriented such that the detector is at 2.5° off-axis location. Super-Kamiokande is a 22.5 kton fiducial mass water Cerenkov detector, capable of good discrimination between electron and muon neutrino interactions [24]. The neutrino flux peaks sharply at 0.7 GeV which is also the energy of the first oscillation maximum. T2K experiment started taking data in 2009 and ran in neutrino mode with 6.6×10^{20} POT till 2013 [6, 18]. Presently they are taking data in anti-neutrino mode.

The ν_e appearance data of T2K were published and analyzed in ref. [18]. They find the

best fit point to be normal hierarchy with $\delta_{\text{CP}} = -90^\circ$. In general both hierarchies with the δ_{CP} values in the lower half plane are allowed at 2σ , whereas δ_{CP} values in the upper half plane are disfavoured for both hierarchies.

From fig. (4) of [18], we get the binned event rates N_i^{data} as a function of reconstructed neutrino energy for electron appearance. Using GLOBES software, we calculated the electron appearance events N_i^{test} for the energy bin i and as a function of the neutrino test parameters $|\Delta m_{\mu\mu}^2|$, $\sin^2 2\theta_{13}$, $\sin^2 \theta_{23}$, δ_{CP} and hierarchy. Then we calculated Poissonian χ^2 as a function of the test parameters using the formula given in eq. 4. The minimum of the χ^2 is obtained and is subtracted from all values of χ^2 s to get $\Delta\chi^2$ as a function of test parameters. This $\Delta\chi^2$ is marginalized over $\Delta m_{\mu\mu}^2$, $\sin^2 \theta_{13}$ and $\sin^2 \theta_{23}$ but not over test δ_{CP} and test hierarchy. We have plotted this $\Delta\chi^2$ in fig. 7 as a function of test δ_{CP} for test hierarchy NH as well as IH.

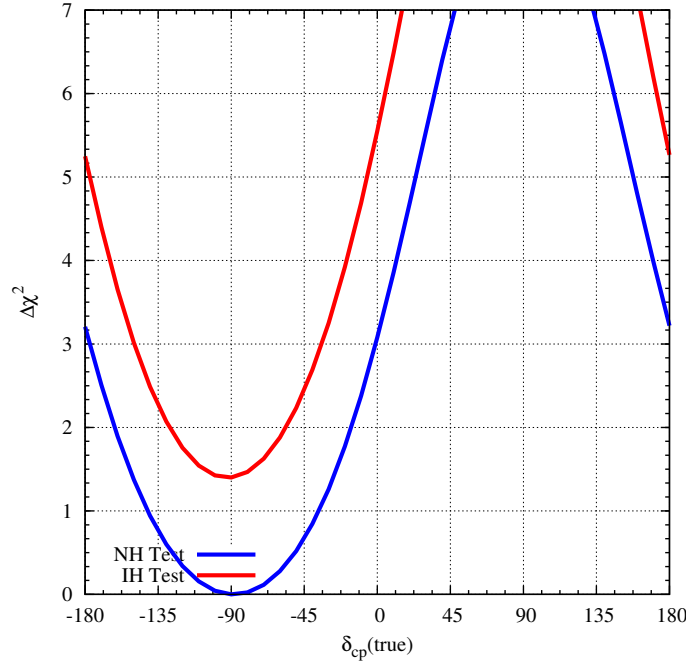


FIG. 7: $\Delta\chi^2$ vs test δ_{CP} plot for T2K neutrino appearance data

C. Combining NO ν A simulations with T2K data

In the next step, we would like to explore how the T2K data modifies the hierarchy determination capability of NO ν A. As described earlier, we have a hundred different sets of

$\chi^2(p)$ ($p = 1, 2, \dots, 100$) each as a function of the test values of neutrino parameters, for the 100 simulations of NO ν A. We also have χ^2 of T2K as a function of the same test values. We now define

$$\chi^2(p)(\text{tot}) = \chi^2(p) + \chi^2(\text{T2K}). \quad (6)$$

In the above addition, we have taken care that the test values of neutrino parameters are the same for both $\chi^2(p)$ and $\chi^2(\text{T2K})$. Note that $\chi^2(p)$ includes the prior coming due to the deviation of the test values of neutrino parameters from their best fit values. From $\chi^2(p)(\text{tot})$ we obtain $\overline{\Delta\chi_m^2}(\text{tot})$ using the same procedure that was used to calculate $\overline{\Delta\chi_m^2}$ from $\chi^2(p)$, that was described in subsection 3.1. This quantity shows how the hierarchy determination capability of NO ν A is modified by the T2K data. To simplify the notation a little, we label this quantity as $\Delta\chi_{\text{HR}}^2$, i.e. the $\Delta\chi^2$ for hierarchy resolution. In the next section, we discuss our results where we have calculated $\Delta\chi_{\text{HR}}^2$ for various different true hierarchy- δ_{CP} combinations.

IV. RESULTS

We have calculated $\Delta\chi_{\text{HR}}^2$ for a number of combinations of true values of hierarchy and δ_{CP} , both favourable and unfavourable. In this section we give the a series of plots of $\Delta\chi_{\text{HR}}^2$ as a function of test δ_{CP} for both of the test hierarchy being the true hierarchy and the test hierarchy being the wrong hierarchy. If $\Delta\chi_{\text{HR}}^2 \geq 4$ for all the values of test δ_{CP} when the test hierarchy is the wrong hierarchy, then the wrong hierarchy can be ruled out at $\geq 95\%$ confidence level. For the cases where this is not true, the hierarchy determination is not possible. We present our results in the following progression.

A. NH as the true hierarchy and true $\delta_{\text{CP}} = -135^\circ, -90^\circ, -45^\circ$

Here all the values of true δ_{CP} are in the lower half plane and hence all the three cases are favourable for the hierarchy determination by NO ν A. Fig. 8 shows the plots for NH (IH) as the test hierarchy in the upper (lower) panels. As we can see, in all the lower panels $\Delta\chi_{\text{HR}}^2 \geq 7$, meaning that the wrong hierarchy can be ruled out quite effectively. We also find, from the lower panels, that the addition of T2K data does not lead to any change in the conclusions one obtains from the simulations of NO ν A. For the upper panels, where the

test hierarchy is the true hierarchy, the minimum value of $\Delta\chi_{\text{HR}}^2 \simeq 2$ is obtained for value of test δ_{CP} in the same half plane as the input value of true δ_{CP} . The non-zero value of minimum $\Delta\chi_{\text{HR}}^2$, as explained in the previous section, arises due to taking the average of a hundred simulations.

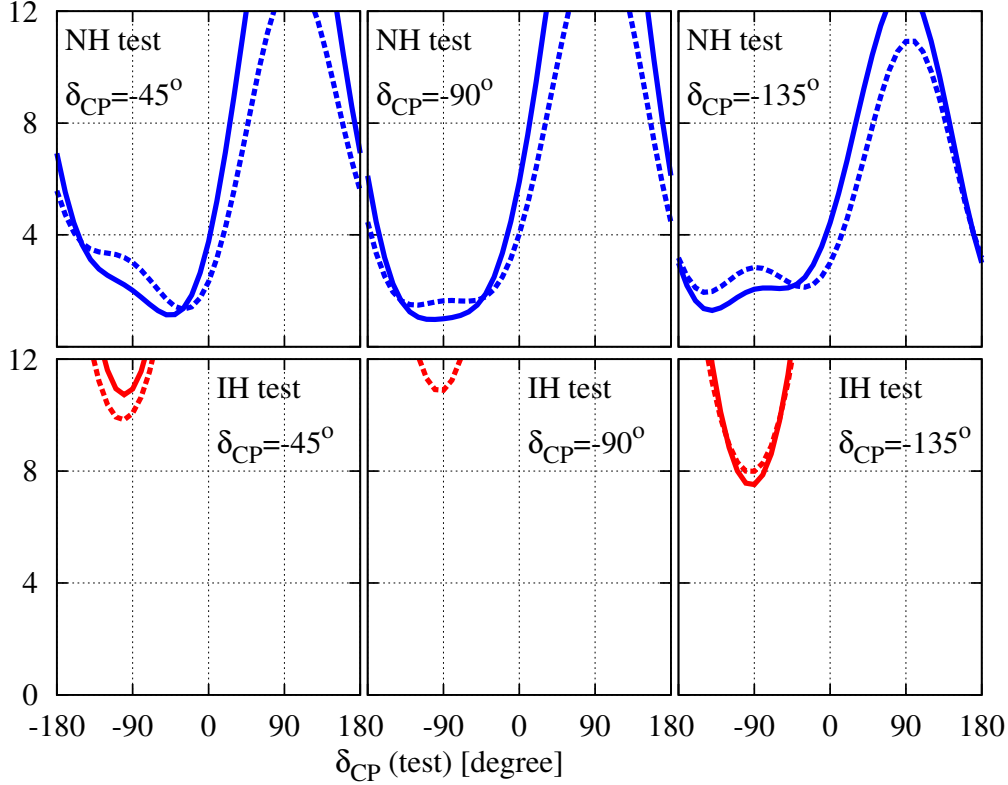


FIG. 8: $\Delta\chi^2$ vs test δ_{CP} plot for NH true and true δ_{CP} in lower half plane. The upper (lower) panel shows the plot for test hierarchy NH (IH). True δ_{CP} values are written on the panels. The solid (dashed) lines give hierarchy determination capability of $\text{NO}\nu\text{A}$ as a function of test values of δ_{CP} , with (without) the addition of T2K data.

B. IH as the true hierarchy and true $\delta_{\text{CP}} = 45^\circ, 90^\circ, 135^\circ$

These three cases are also favourable for the hierarchy determination by $\text{NO}\nu\text{A}$ alone. Fig. 9 shows the plots for NH (IH) as the test hierarchy in the upper (lower) panels. Here we find that the $\Delta\chi_{\text{HR}}^2 \geq 9$ in all the upper panels which means that the wrong hierarchy can be ruled out at nearly 3σ level. Looking at the lower panels, we find a minimum

$\Delta\chi_{\text{HR}}^2$ of about 1 close to test $\delta_{\text{CP}} \sim 30^\circ$. This occurs because of the clash between the NO ν A simulation and T2K data. T2K data disfavours IH and δ_{CP} in the upper half plane. In fact, the point IH- $\delta_{\text{CP}} = 90^\circ$ has a $\Delta\chi^2 = 6$ from the T2K data. However, in our calculations, we obtain a lower $\Delta\chi_{\text{HR}}^2$ for test hierarchy IH and test δ_{CP} in the upper half plane when NO ν A simulation is combined with T2K data due to the following reason. The point favoured by NO ν A simulation is disfavoured by T2K data and vice versa. Therefore the combination of the two has a minimum $\Delta\chi_{\text{HR}}^2$ at some intermediate point. The reason why the points with IH and test δ_{CP} in upper half plane are not disfavoured by the combined data is because the hierarchy discrimination capability of the full run of NO ν A outweighs the corresponding discrimination of the current T2K neutrino run. Hence these points, if they happen to be the true points, will be favoured by NO ν A (and by NO ν A plus T2K) even though they are presently disfavoured by T2K.

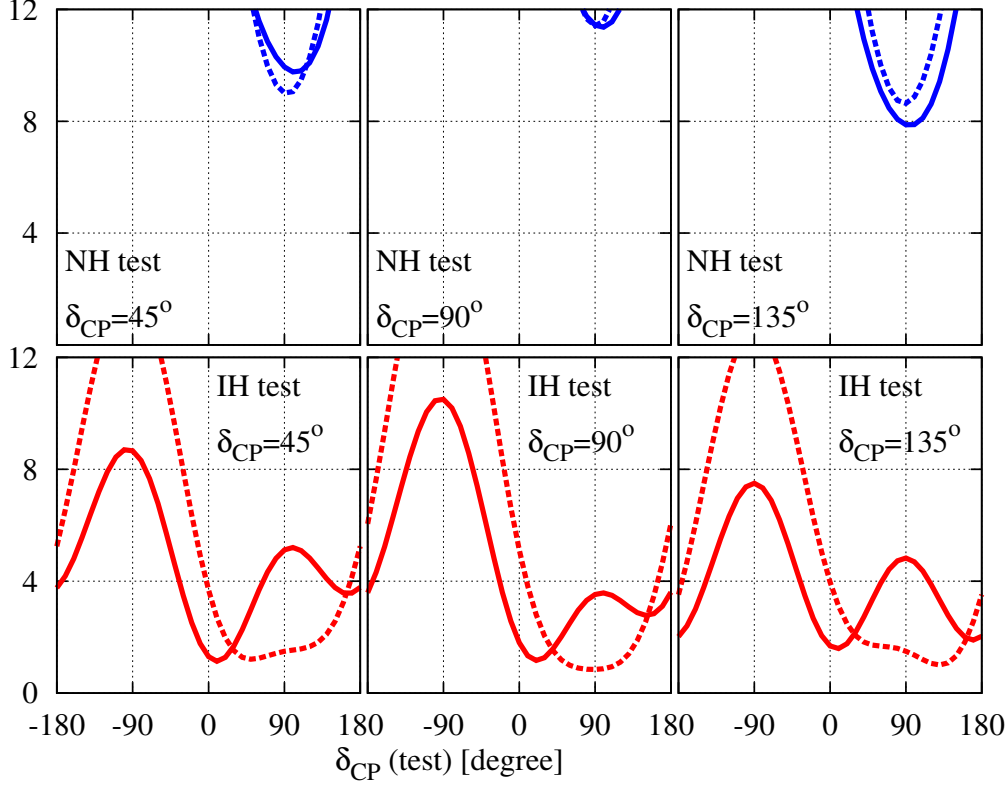


FIG. 9: $\Delta\chi^2$ vs test δ_{CP} plot for IH true and true δ_{CP} in upper half plane. The upper (lower) panel shows the plot for test hierarchy NH (IH). True δ_{CP} values are written on the panels. The solid (dashed) lines give hierarchy determination capability of $\text{NO}\nu\text{A}$ as a function of test values of δ_{CP} , with (without) the addition of T2K data.

C. NH as the true hierarchy and true $\delta_{CP} = 45^\circ, 90^\circ, 135^\circ$

These hierarchy- δ_{CP} combinations are unfavourable for hierarchy determination by $\text{NO}\nu\text{A}$ alone. If these are the true combinations, the fit to $\text{NO}\nu\text{A}$ data yields two degenerate solutions: One with the NH and δ_{CP} in upper half plane and one with IH and δ_{CP} in lower half plane. The $\Delta\chi^2$ of $\text{NO}\nu\text{A}$ simulations for these solutions will be the same. If we add the T2K data, which disfavours δ_{CP} in upper half plane, we find that the true solution of NH and δ_{CP} in the upper half plane has a rather large $\Delta\chi_{\text{HR}}^2 \geq 4$ whereas the wrong hierarchy solution, IH with δ_{CP} in the lower half plane, has $\Delta\chi_{\text{HR}}^2 \leq 4$. This can be seen in fig. 10, where $\Delta\chi_{\text{HR}}^2$ vs test δ_{CP} is plotted for test hierarchy NH (IH) in upper (lower)

panel.

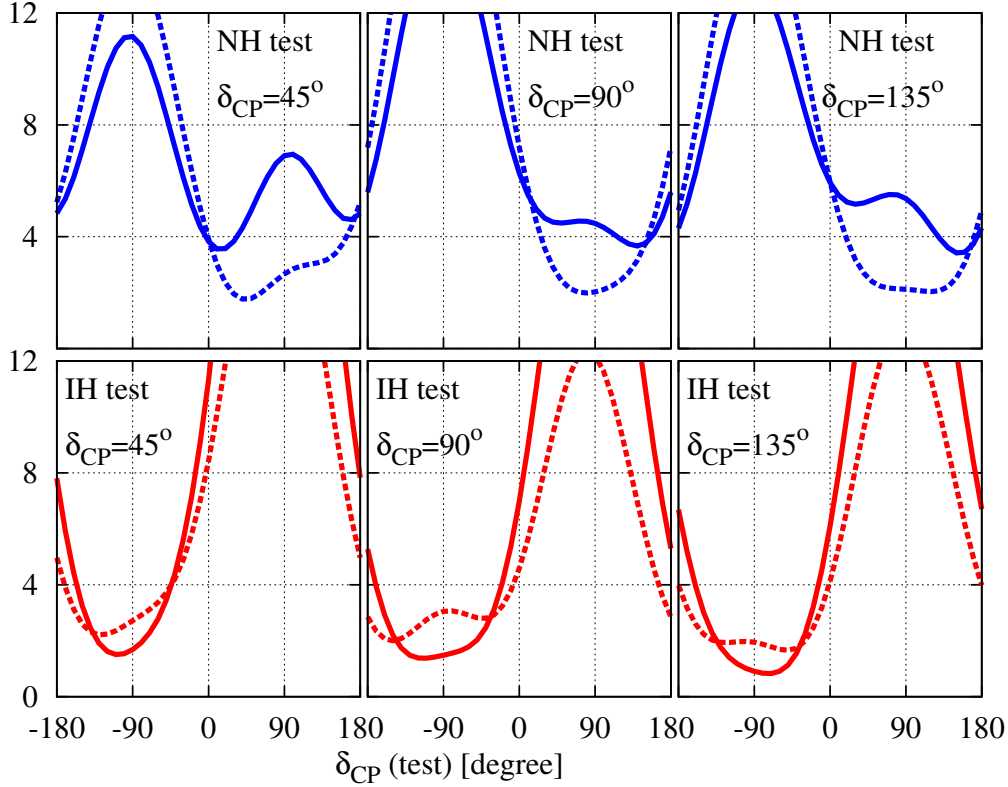


FIG. 10: $\Delta\chi^2$ vs test δ_{CP} plot for NH true and true δ_{CP} in upper half plane. The upper (lower) panel shows the plot for test hierarchy NH (IH). True δ_{CP} values are written on the panels. The solid (dashed) lines give hierarchy determination capability of $\text{NO}\nu\text{A}$ as a function of test values of δ_{CP} , with (without) the addition of T2K data.

D. IH as the true hierarchy and true $\delta_{CP} = -45^\circ, -90^\circ, -135^\circ$

These are also unfavourable hierarchy- δ_{CP} combinations for hierarchy determination by $\text{NO}\nu\text{A}$. For this case also, we will have degenerate solutions of NH with δ_{CP} in the upper half plane and IH with δ_{CP} in the lower half plane. Here the addition of T2K data picks out the correct solution of IH with δ_{CP} in the lower half plane. The hierarchy determination plots are shown in fig. 11 with test hierarchy NH (IH) in upper (lower) panel. We see from this plot that for NH test, $\Delta\chi^2_{\text{HR}} > 4$ for all test values of δ_{CP} . Thus addition of T2K data with $\text{NO}\nu\text{A}$, helps to exclude the wrong hierarchy at 2σ .

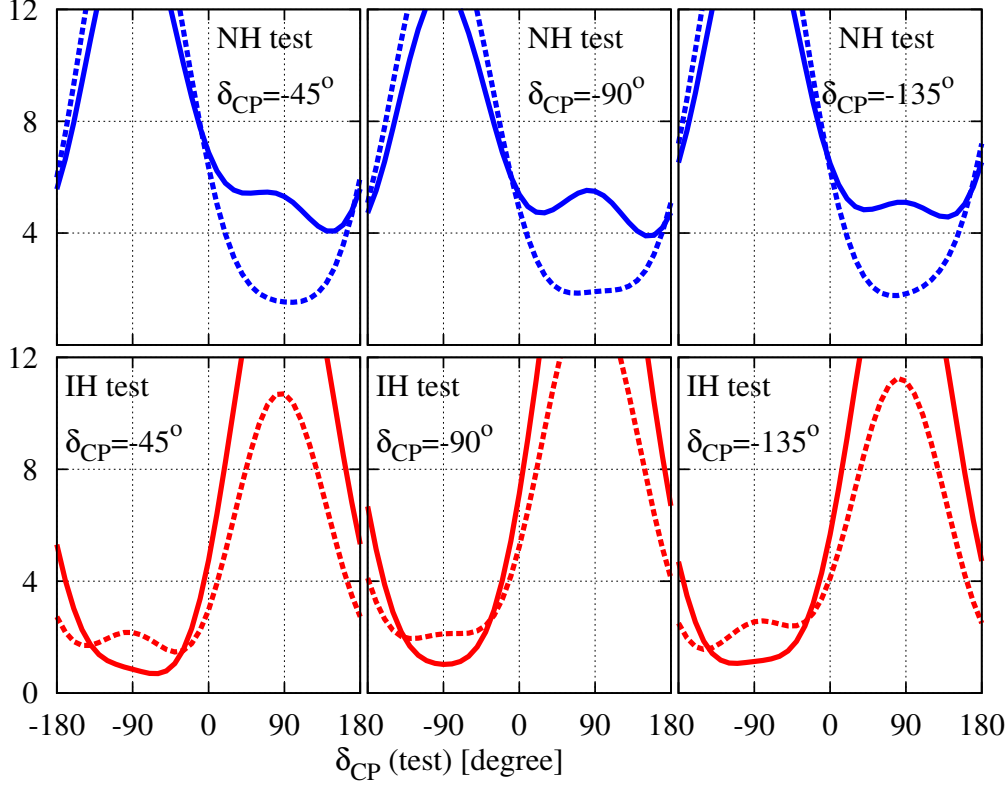


FIG. 11: $\Delta\chi^2$ vs test δ_{CP} plot for IH true and true δ_{CP} in lower half plane. The upper (lower) panel shows the plot for test hierarchy NH (IH). True δ_{CP} values are written on the panels. The solid (dashed) lines give hierarchy determination capability of $\text{NO}\nu\text{A}$ as a function of test values of δ_{CP} , with (without) the addition of T2K data.

E. Hierarchy determination for true $\delta_{\text{CP}} = 0, 180^\circ$

These are the CP conserving δ_{CP} values for NH true. Fig. 12 shows the plots with true $\delta_{\text{CP}} = 0$ (180°) in left (right) panel and test hierarchy NH (IH) in upper (lower) panel. From the figure we see that for both the CP conserving δ_{CP} values, the wrong hierarchy can not be excluded completely at 2σ C.L., even after the addition of T2K data with $\text{NO}\nu\text{A}$. Thus hierarchy determination is not possible for the CP conserving values of δ_{CP} when NH is the true hierarchy. However, when IH is the true hierarchy, NH can be effectively ruled out for the CP conserving δ_{CP} values, as illustrated in fig. 13.

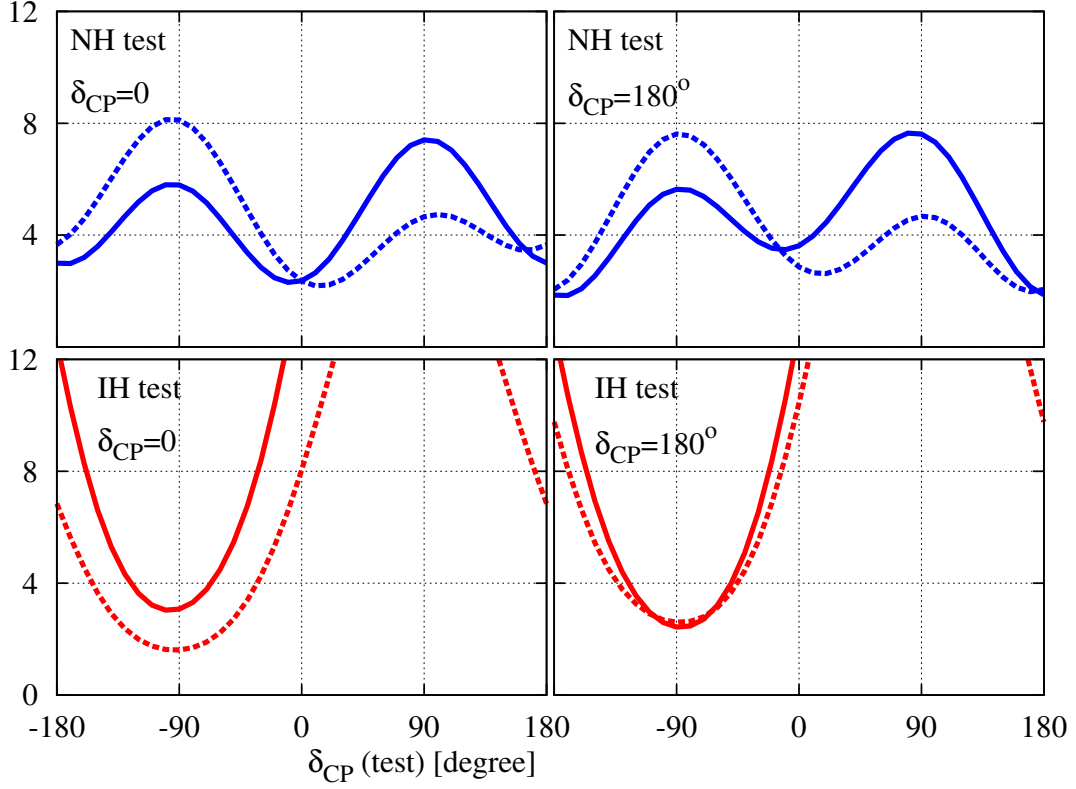


FIG. 12: $\Delta\chi^2$ vs test δ_{CP} plot for NH true and true δ_{CP} with CP conserving values. The left (right) plot is for true value of $\delta_{\text{CP}} = 0$ (180°). Test hierarchy is NH (IH) for top (bottom) panel. The solid (dashed) lines signify $\text{NO}\nu\text{A}$ simulations combined with (without) T2K data.

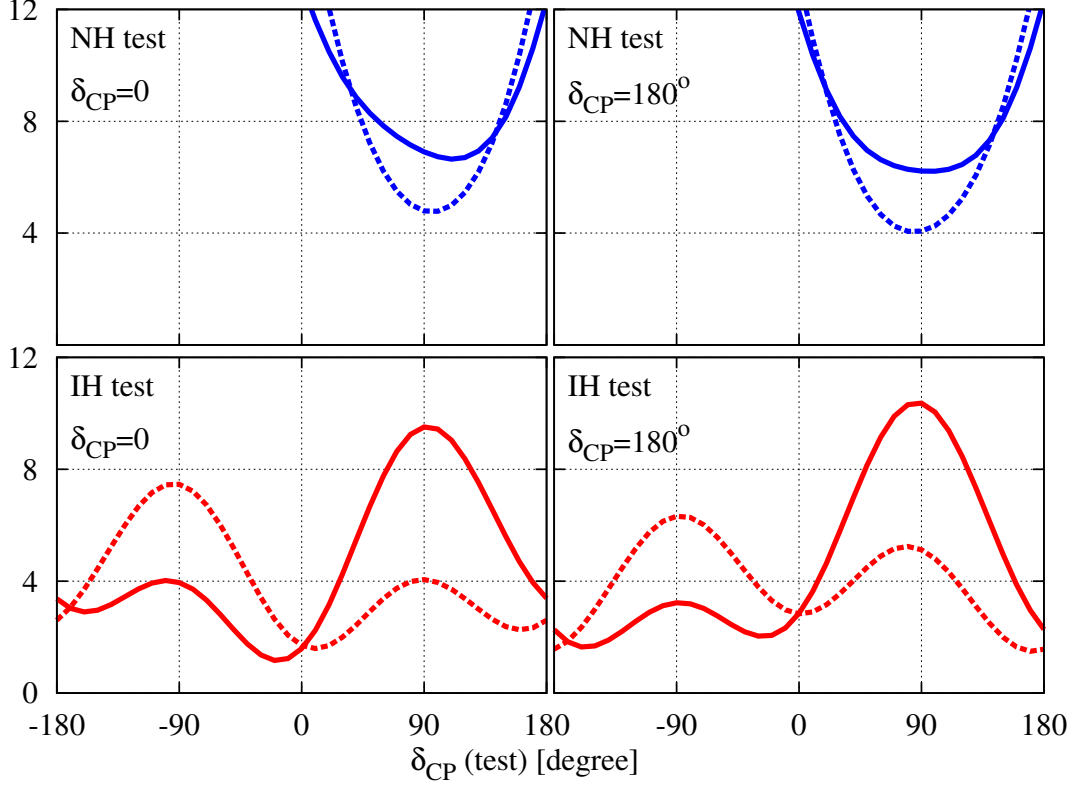


FIG. 13: $\Delta\chi^2$ vs test δ_{CP} plot for IH true and true δ_{CP} with CP conserving values. The left (right) plot is for true value of $\delta_{CP} = 0$ (180°). Test hierarchy is NH (IH) for top (bottom) panel. The solid (dashed) lines signify NO ν A simulations combined with (without) T2K data.

V. ANALYSIS OF RECENT NO ν A AND T2K DATA

In the previous section, we studied the effect of combining the ν_e appearance data of T2K [18] with NO ν A simulations to estimate the hierarchy determination potential. Recently, T2K has published their anti-neutrino data corresponding to an exposure of 4×10^{20} POT [26] and NO ν A has released the results of their first neutrino run with an exposure of 2.7×10^{20} POT [27, 28]. It will be interesting to study the neutrino parameter space allowed by these three pieces of data.

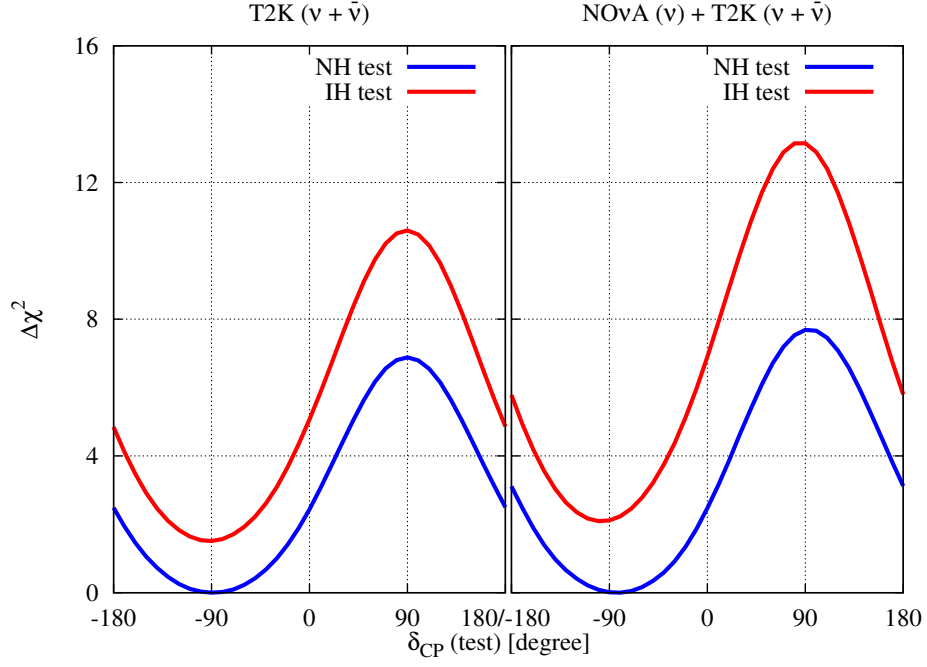


FIG. 14: $\Delta\chi^2$ vs test δ_{CP} plot for combined analysis of T2K and NO ν A data. The left (right) panel shows the analysis of T2K neutrino and anti-neutrino appearance data without (with) the NO ν A neutrino appearance data.

In fig. 14, we have shown $\Delta\chi^2$ from the combined appearance data of T2K ν and $\bar{\nu}$ runs and NO ν A ν run, as a function of test values of δ_{CP} for both NH and IH as test hierarchies. The results in this plot show the same features as the results obtained from the analysis of T2K neutrino data. The best fit point occurs for NH and $\delta_{\text{CP}} = -90^\circ$. For both the hierarchies, the lower half plane is favoured and the upper half plane is disfavoured. In particular, a large fraction of the upper half plane is ruled out at 2σ for NH and the whole of it ruled out at 2σ for IH. Our results match with those of ref. [29] obtained earlier.

VI. SUMMARY AND CONCLUSIONS

In this paper we have studied influence of the present neutrino data of T2K on the hierarchy determination ability of NO ν A. This study required combining the simulations of NO ν A with the data of T2K. This posed a challenge because fluctuations are inherent in the data of T2K. We overcame this problem by simulating the NO ν A data with Poissonian fluctuations. To minimize the effect of the fluctuations, we did 100 different simulations and

took the average. We also showed that a larger number of simulations do not change our conclusions.

Regarding the hierarchy determination capability of $\text{NO}\nu\text{A}$, T2K data has no effect if the hierarchy- δ_{CP} combinations are favourable. For such cases, $\text{NO}\nu\text{A}$ data determines the hierarchy. For the unfavourable combinations one must exercise care. For the combination IH and δ_{CP} in lower half plane, the T2K data picks out the correct solution between the degenerate solutions allowed by the $\text{NO}\nu\text{A}$ data. For the combination NH and δ_{CP} in the upper half plane, the T2K data favours the wrong hierarchy-wrong δ_{CP} solution between the degenerate solutions. If the combination of T2K and $\text{NO}\nu\text{A}$ data gives IH and δ_{CP} in the lower half plane as the preferred solution, it may not be correct. It is possible that the correct solution is NH and δ_{CP} in the upper half plane but the preference of the present T2K neutrino appearance data for δ_{CP} in the lower half plane leads to the wrong solution. Hence we conclude that the present neutrino data of T2K does not help in rejecting the wrong hierarchy, in the case of unfavourable combinations. In such a situation, data from an experiment such as DUNE [30] is needed to resolve the hierarchy- δ_{CP} degeneracy.

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